Why Do Shallow Water Waves “Move Faster in Deep Water and Slower in Shallow Water”?

(Here, “deep water” and “shallow water” refer to different depths that are both much smaller than the wavelength.)

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This note serves as an addendum to my article published in *Physics Education*, “**Revisiting the simple model of the late Professor Frank S. Crawford for water wave dispersion relations, and thus obtaining a physical explanation of the depth-dependence of shallow water wave speeds.**”
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It aims to elaborate on two specific aspects.
First, additional arguments are presented to further substantiate that bottom friction cannot account for the observed phenomenon in which shallow water waves travel slowly.
Secondly, a simple and practical experimental method is introduced that can reproduce part of the conclusions reached in the original article.

Nevertheless, this note may also be read as a self-contained piece.

Why “Bottom Friction” Is Not the Correct Explanation

1. Bottom friction not always opposes wave’s motion

For shallow waves, *water particles right at the bottom oscillate back and forth in periodic motion parallel to the floor*.

If bottom friction exists, it resists motion half the time and assists it the other half, alternating in direction. Clearly, this cannot systematically slow the entire wave, totally unlike the case of a block sliding across a rough surface.

2. Formula does not contain “bottom friction”

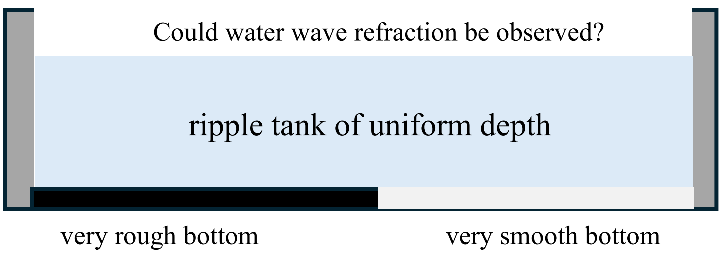
- The verified relation $c = \sqrt{gd}$ follows from well-established fluid theory and experiments, with no friction term present.

- If bottom friction truly caused the speed difference, one should be able to derive the same formula from bottom frictional forces—yet no such derivation exists.

- Moreover, friction would only slow waves, never speed them up. But as waves move from deep to shallow and back to deep regions, their speed decreases and then increases again; a phenomenon bottom friction alone cannot explain.

3. Bottom friction has very small effect on wave speed

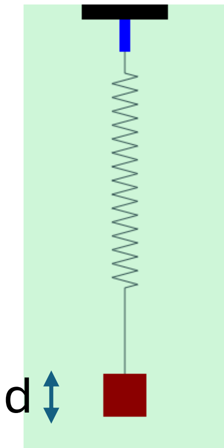
- If friction significantly affected wave speed, one could observe refraction by simply altering the roughness of the tank bottom at constant depth. This does not occur in experiments, implying friction’s influence on wave speed is negligible.



- In hydrodynamics, bottom friction primarily causes **amplitude attenuation**, not significantly reducing wave speed.

4. A Mechanical Analogy

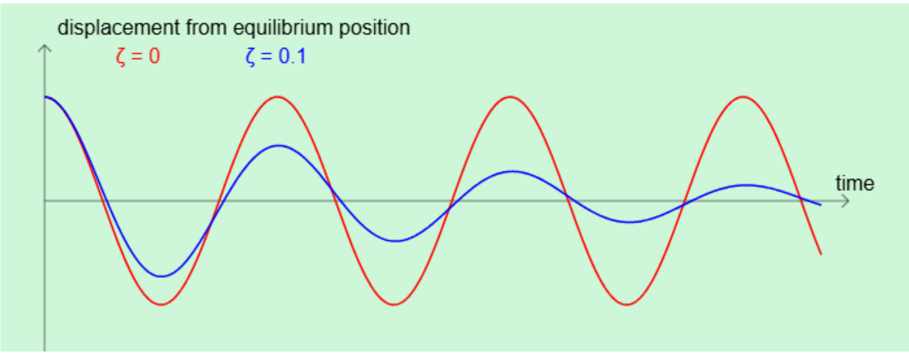
Imagine a spring suspending a solid cube of height d. Pull it down slightly and release. This system performs simple harmonic motion.



Question: Why does the oscillation frequency decrease when d increases?

Answer: A larger cube implies an increased cross-sectional area, resulting in higher air resistance while oscillating. Consequently, its motion is impeded to a greater extent, leading to a reduction in oscillation frequency.

Is this answer correct? Definitely not; although damping, in fact, can slightly reduce frequency, it cannot explain the actual drop.



For instance, when *d* is doubled, the mass *m* increases eightfold, and the frequency decreases by a factor of 0.35, of which the air resistance can in no way be explained. The correct descriptive formula is $\omega = \sqrt{k/m}$.

Air resistance changes frequency very slightly, but it is not the *fundamental* cause.

Likewise, *bottom friction might somehow reduce wave speed very slightly but surely is not the basic reason that "shallower water waves move slower."*

The Center-of-Mass Simple Pendulum Model

In 1987, UC Berkeley physicist **Frank S. Crawford** published a short paper in *American Journal of Physics* titled *"A simple model for water-wave dispersion relations." (Am. J. Phys. 55, 171–172 (1987))* He showed that one can derive $c = \sqrt{gd}$ by modeling a water *standing wave* as the motion of the system's **center of mass (CM)**.

We find this model not only reproduces the formula but also provides an intuitive qualitative reason for the depth-dependence of shallow water wave speeds"

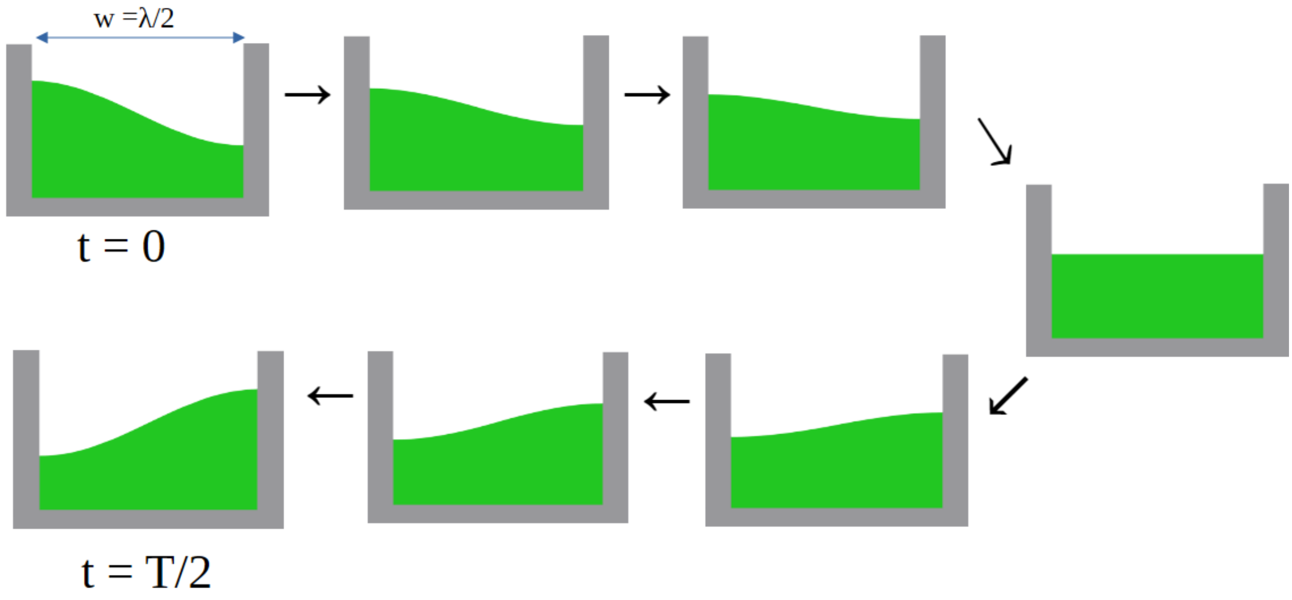
Consider a rectangular tank filled with water in which a **standing wave** is formed.

1. The wavelength λ equals twice the tank width w: $\lambda = 2w$, representing the fundamental mode.

2. Neglect friction in the water and at the boundaries; both ends are antinodes.

3. Wave speed $c = \lambda/T$, where T is the period.

4. In everyday experience, when you carry a basin of water that sloshes back and forth, you are witnessing this very oscillation.



We now ask two questions:

1. How does the water body’s **center of mass** move as the standing wave oscillates?

2. How does this motion change with increasing water depth?

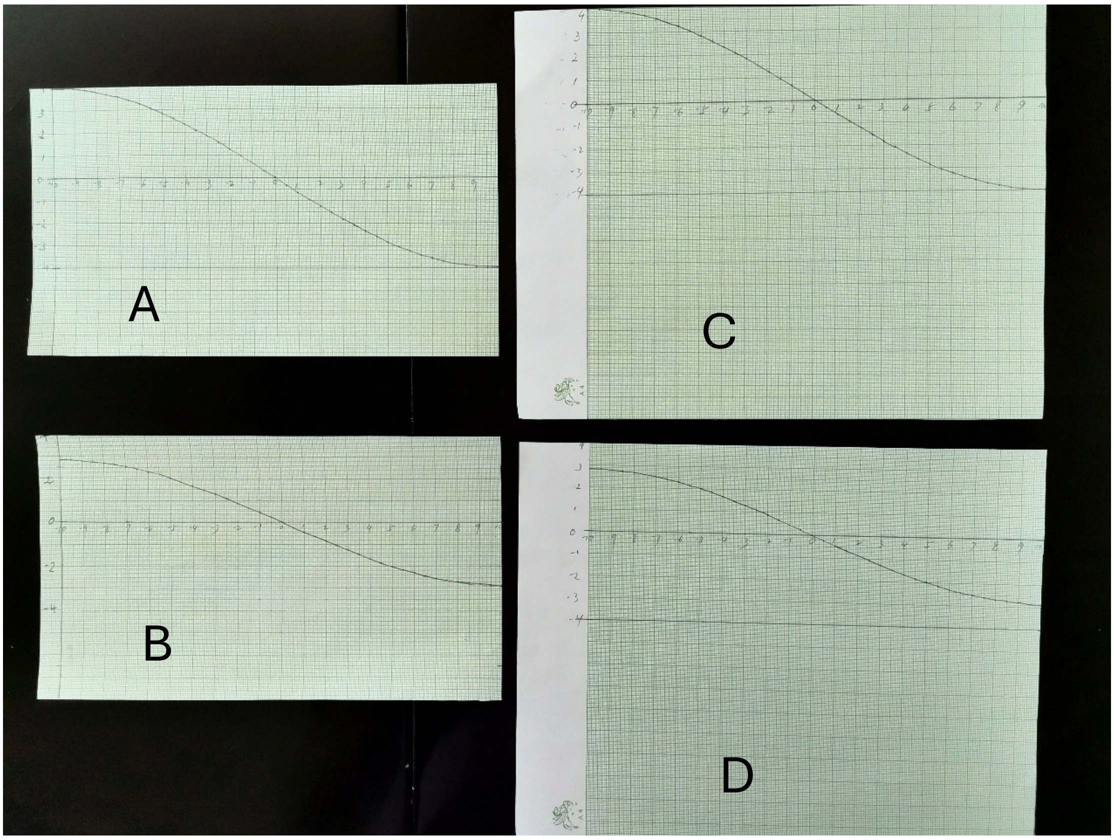
Before reading on, try predicting the outcome.

1. A hand-on experiment

The "Plumb Line Method" is employed to locate the center of mass of the standing wave.

Step 1: On graph paper, plot the wave profile

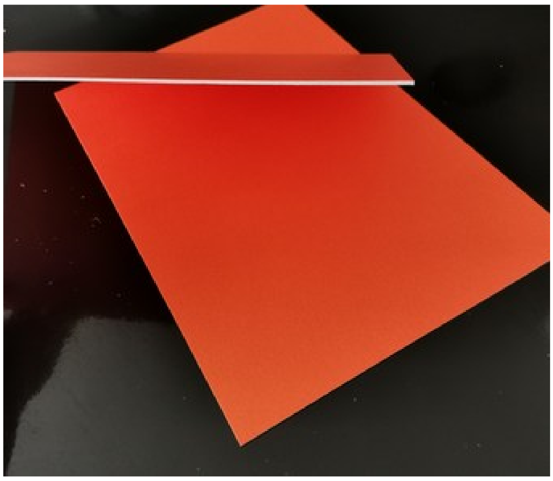
$y = A\cos\left(2\pi\frac{x}{\lambda}\right)\cos\left(2\pi\frac{t}{T}\right)$ for two different water depths at several instants in time (for example, two).



Curve parameters

- All Graphs: $\lambda = 40$ units, $A = 4$ units. (1 unit = 1 cm)
- Graph A & B: $d = 8$ units (shallow water)
- Graph C & D: $d = 14$ units (deep water)
- Graph A & C: t = 0
- Graph B & D: t = T/8

Step 2: Mount the graph papers on stiff uniform card boards.

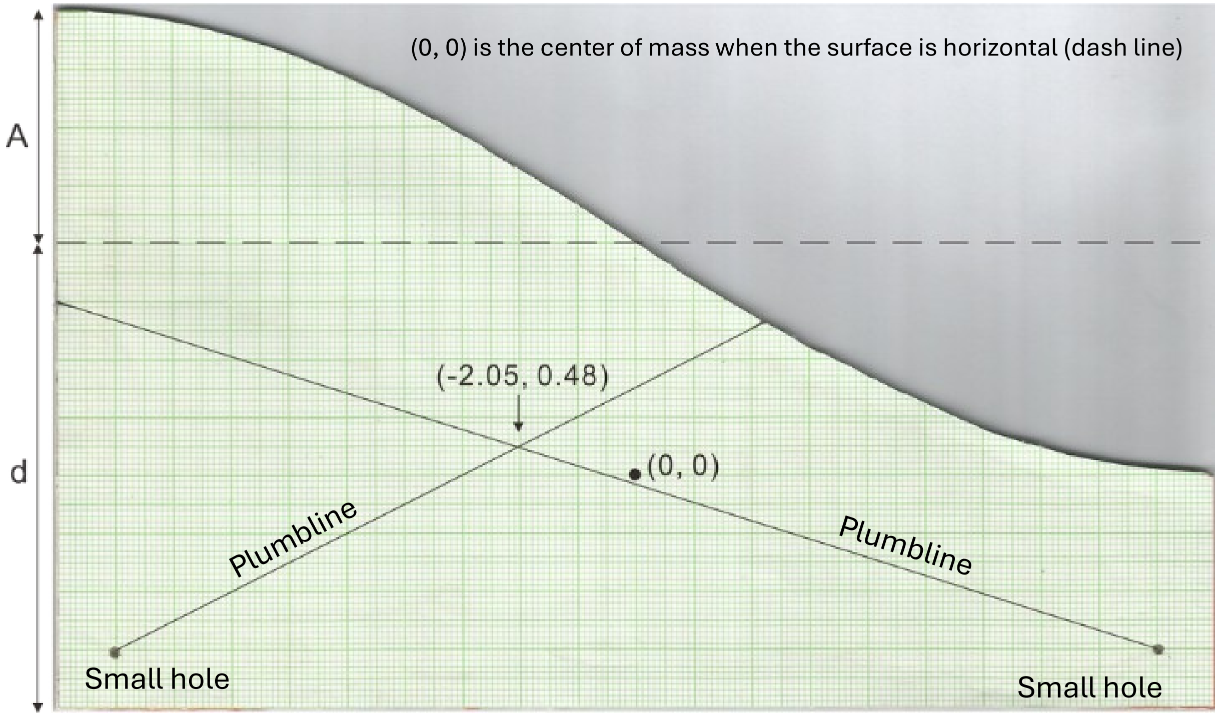


Step 3: Cut out the wave shape.

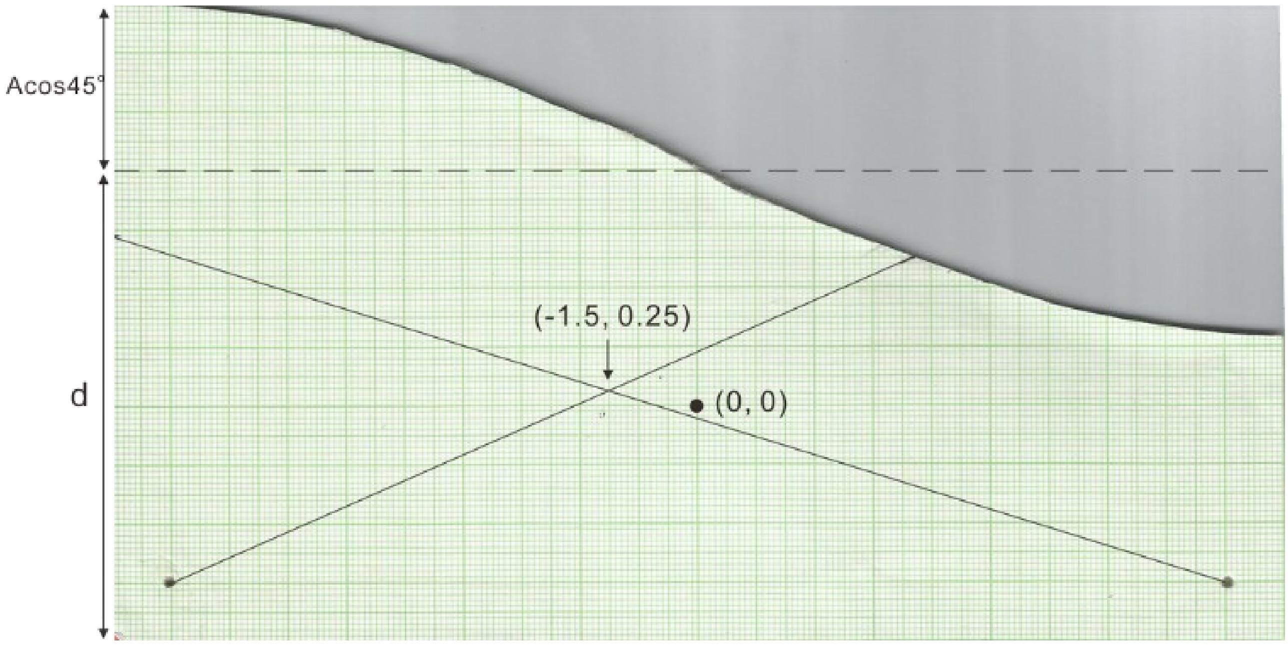
Step 4: Use two small holes and a plumb line to locate the CM for each shape.

2. Experimental Results:

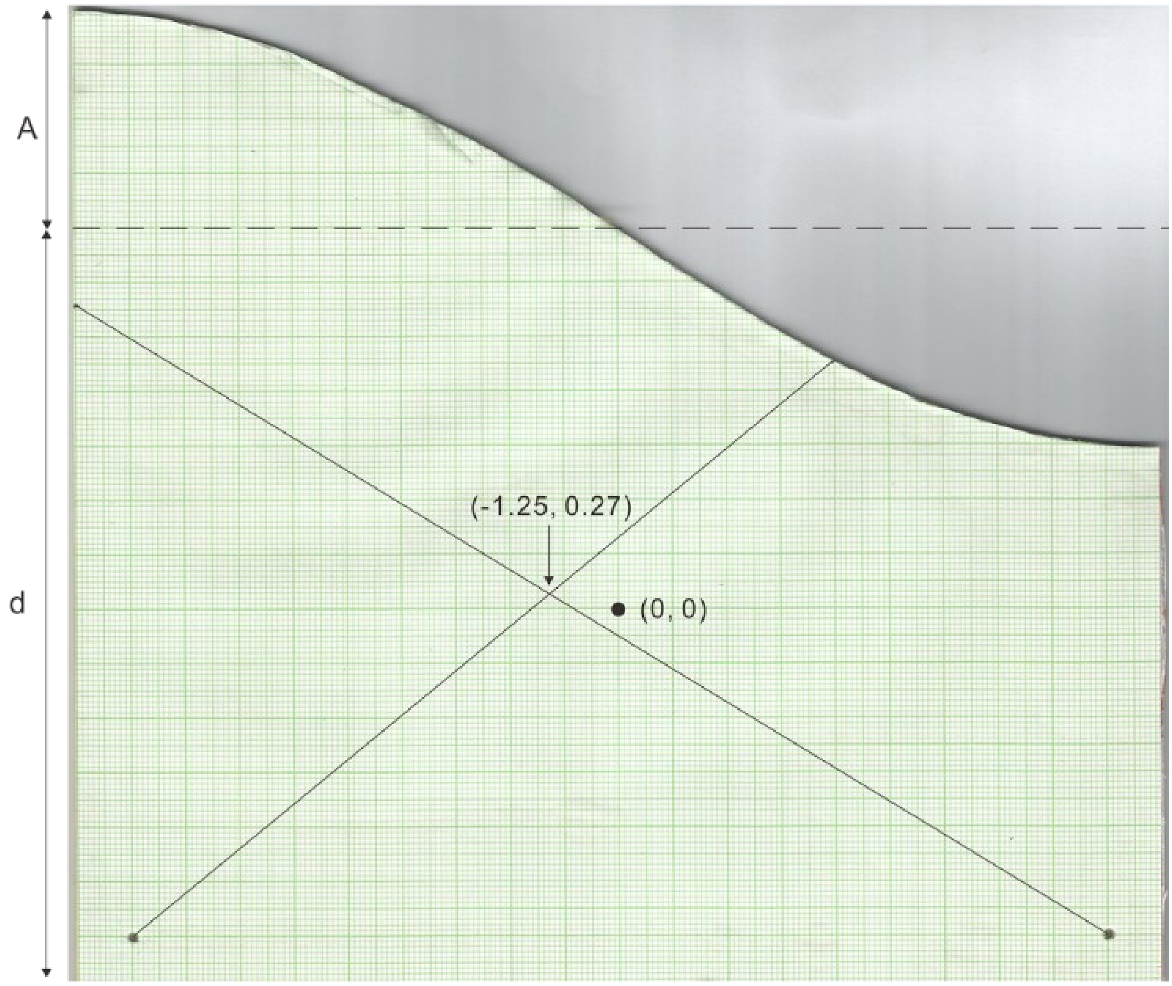
(i) Shallow water (t = 0)



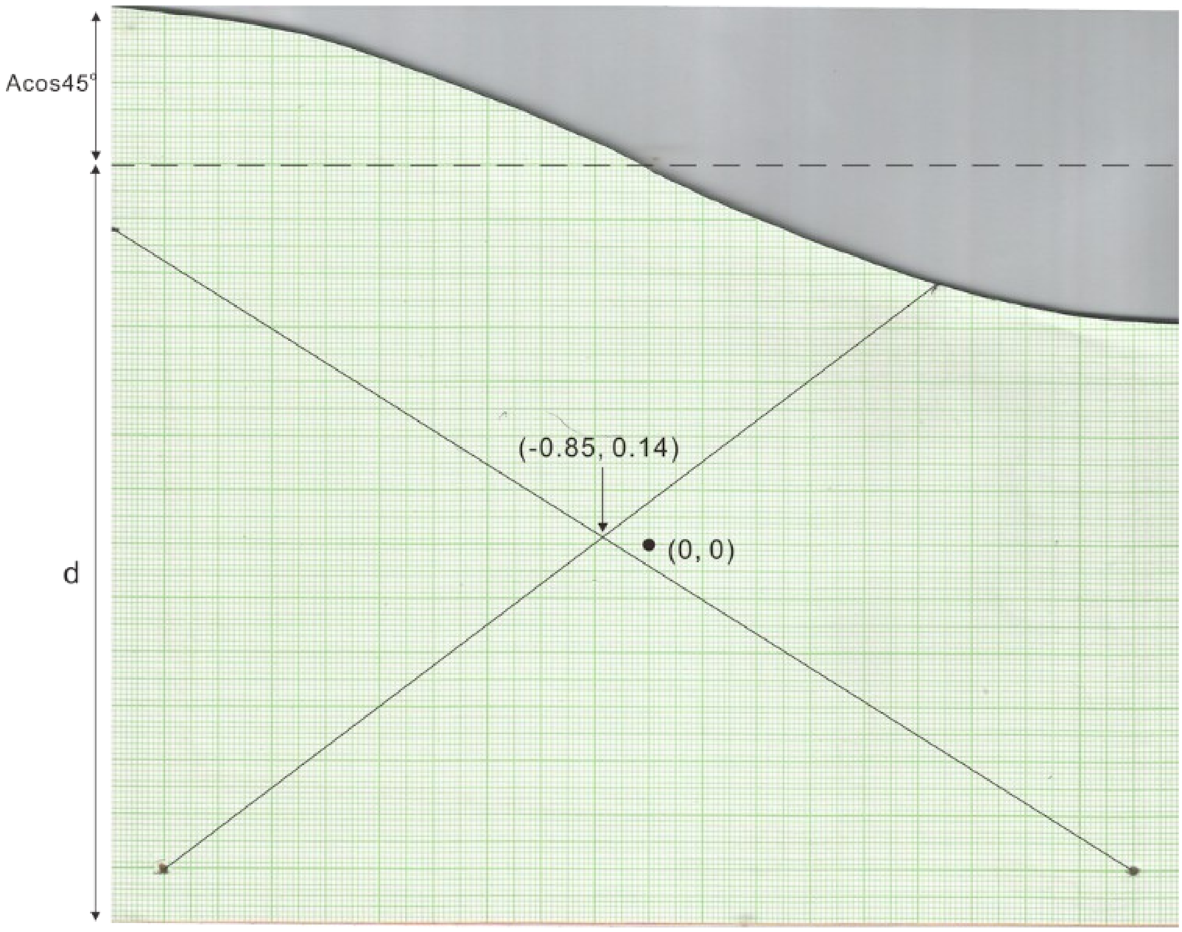
(ii) Shallow water (t = T/8)



(iii) Deep water (t = 0)



(iv) Deep water (t = T/8)



3. Graph plotting

Graph 1 (Shallow water):

λ = 40 units, A = 4 units, d = 8 units.

CM's Locus

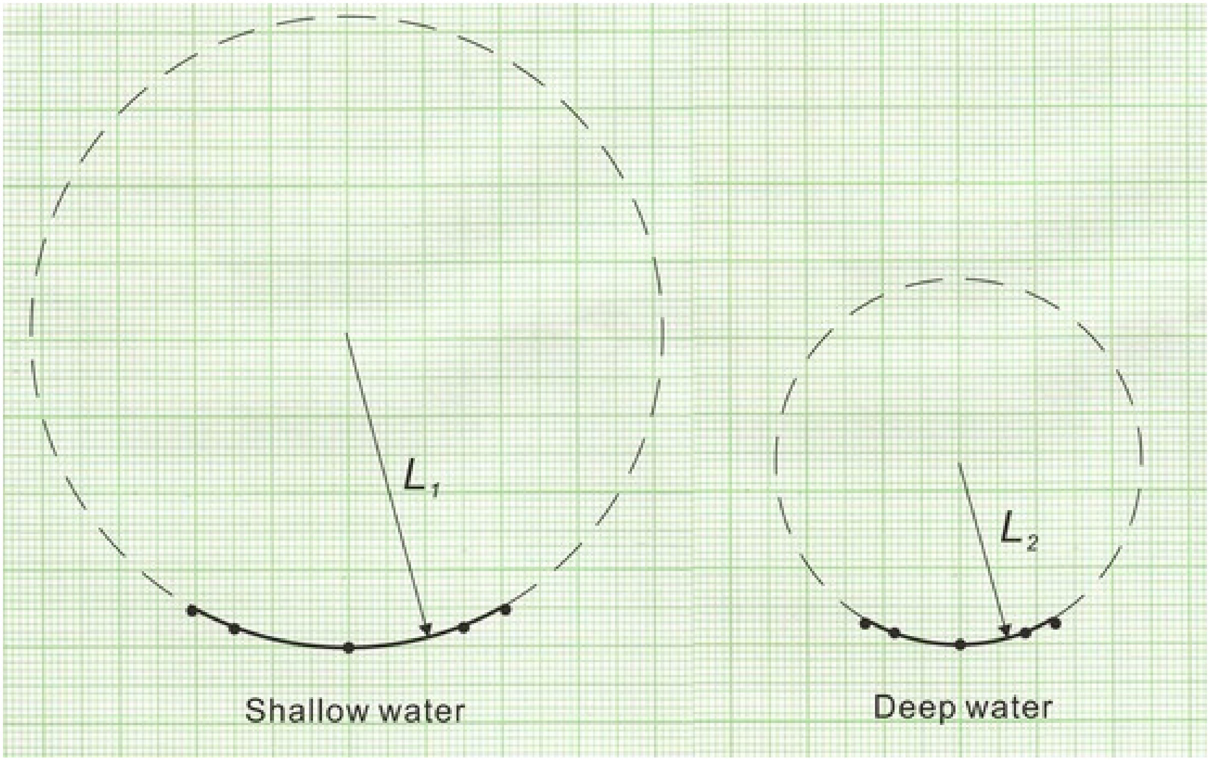
x_{CM}	-2.05	-1.5	0	1.5	2.05
y_{CM}	0.48	0.25	0	0.25	0.48

Graph 2 (Deeper water):

λ = 40 units, A = 4 units, d = 14 units.

CM's locus

x_{CM}	-125	-0.85	0	0.85	1.25
y_{CM}	0.27	0.14	0	0.14	0.27



4. Results and Interpretation

1. The CM of the standing wave moves in a **circular path**, performing a **simple harmonic motion** governed by $\cos(\omega t)$.

2. Since water waves are driven by gravity, this circular motion effectively behaves like a **simple pendulum**.

3. The period of a pendulum is $T = 2\pi\sqrt{L/g}$, depending only on gravitational acceleration g and pendulum length L.

4. The deeper-water case shows a **shorter effective pendulum length**, hence a shorter period. For the same wavelength, a shorter period implies higher wave speed $c = \lambda/T$.

5. Accurate geometric analysis confirms that $L \propto 1/d$, implying $c \propto \sqrt{d}$.

5. Physical Interpretation

1. Increasing water depth adds more mass beneath the oscillating surface. The deeper the water, the less influence the surface elevation exerts on the overall motion.

2. Thus, deeper water results in smaller particle displacements and a CM that moves in tighter, smaller-radius paths around its equilibrium.

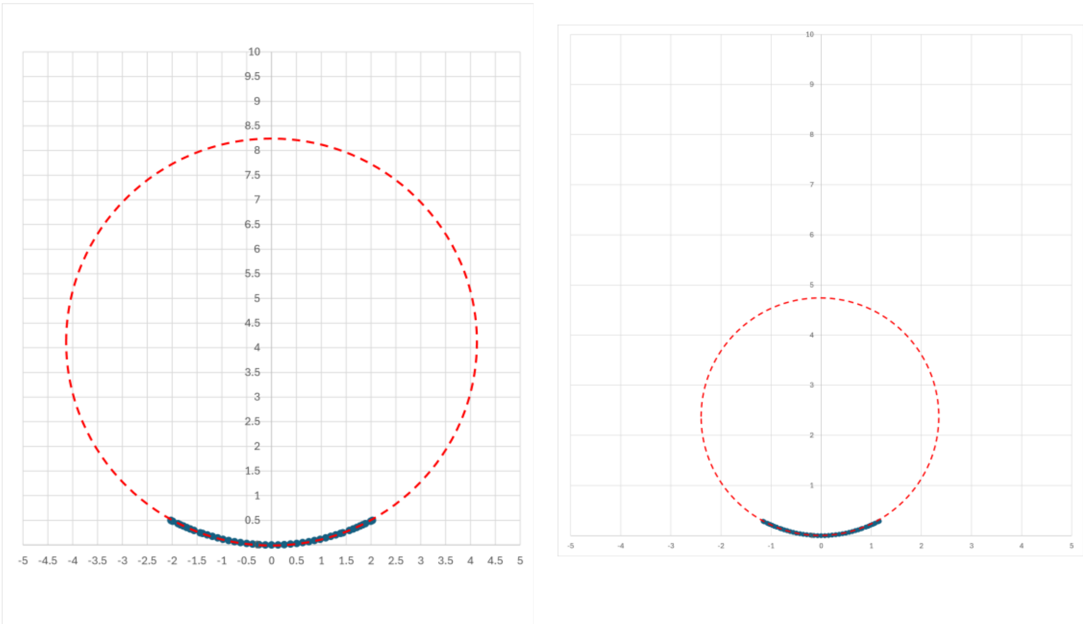
3. The deeper the water, the more the CM path *curves*, effectively shortening the pendulum length, which yields a shorter period and hence faster wave speed.

In short, surface undulations (the "disturbing" effect) and the underlying water mass (the "stabilizing" effect) compete with each other.

As water depth increases, the stabilizing influence dominates, leading to a more curved CM trajectory and faster oscillation.

Discussion

1. The center of mass of the object can even be calculated using a spreadsheet, and the steps are quite simple. The figure below shows a graph produced by us using a spreadsheet. On the left is the shallow-water region, and on the right is the deep-water region. The parameters such as wavelength and water depths are the same as before



When cutting the paper card to represent the water surface shape, precision is not critical; the experimental results remain fairly accurate. What truly matters is that the paper's thickness is uniform and that care is taken during the experiment.

As long as the depths of the two datasets are not too close, the curvature radii of their trajectories will differ significantly, and the answer will become apparent.

2. This "center of mass–pendulum" explanation is a macroscopic one. We are not concerned with the motion of individual particles, but rather with their collective behavior. The advantage of this macroscopic explanation lies in the following:

- There is no need to know the actual motion of individual particles; it is sufficient to obtain the desired result by knowing that the water surface follows a cosine curve.

- The wave speed itself is a macroscopic quantity, and thus explaining it through a macroscopic theory is most appropriate.

- Apart from the "bottom friction" explanation, another popular view holds that the faster wave speed in deep water is due to an increase in the amount (and therefore weight) of water, which supposedly makes it easier to move, thus accelerating the wave. However, "water weight" is also a macroscopic quantity. Therefore, the "center of mass–pendulum" interpretation is most qualified to speak on this matter. This alternative explanation is clearly incorrect, because the period of a simple pendulum is independent of its mass m.

3. The phenomenon is not related to "bottom friction," "water weight," or "water inertia." So what, then, is it related to? The "center of mass–pendulum" model provides a clear, though perhaps not unique, answer: it arises from a geometric cause.
   When the center of mass moves along a flatter or more curved trajectory, the vertical component of gravity, *g*, acts differently in generating oscillations, thereby directly influencing wave propagation.

4. The above analysis uses standing waves on water. Then, is this "center of mass–pendulum" explanation also applicable to traveling waves?
   The simple answer: Yes, it is applicable.
   In the case of water waves that travel, the "pendulum" still exists, though it is not as apparent as in standing waves.

5. Earlier, we assumed that the wave amplitude is the same in both shallow and deep-water regions; this was only for convenience (to be explained later). In fact, in the diagram on page 14, the radius L of the "center-of-mass pendulum" is independent of the amplitude. Readers can produce additional diagrams to experimentally verify this. However, L is related to the wavelength λ. Since the wave speed $c = \lambda / T$, and if we assume that λ remains the same, then any change in c is entirely reflected in the change in T (the period).

6. To our knowledge, apart from this macroscopic explanation, there also exists a microscopic one. It studies how, based on the motion of individual water particles, water is transported from a rising crest through the channel above the seabed toward a depressed trough, thereby deriving the relationship between wave speed and water depth.

   The region between the water surface and the seabed acts like a water pipe: shallow (or deep) water regions correspond to narrow (or wide) pipes, respectively. Hence, this model is commonly called the "water-pipe model."

7. We firmly believe that the "center of mass–pendulum" explanation is correct. Whether it is the best explanation may be subjective, but at the very least, we feel it is highly suitable for secondary school students, because they do not need to master the patterns of motion of water particles beneath the surface. It is enough to know that the water surface takes the shape of a cosine curve.

   Through hands-on experimentation, one can find the result without involving any wave theory at all. The explanation is also easy for teachers to present.

   Some might say, "But secondary students might not even understand what a pendulum motion is!"

   In fact, students don't need a deep understanding. The teacher can demonstrate a simple pendulum experiment, allowing them to observe that the period is independent of amplitude, and that a shorter pendulum oscillates faster.

   Do teachers or parents still tell the story of Galileo observing the swinging lamp in the Pisa Cathedral today?

End