Oblique and Head-On Elastic Collisions

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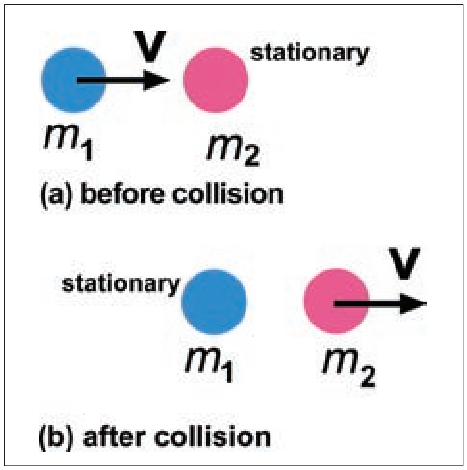
When a moving ball collides elastically with an identical, initially stationary ball, the incident ball will either come to rest (head-on collision; see Fig. 1) or will acquire a velocity that is perpendicular to that acquired by the target ball (oblique collision; see Fig. 2). These two possible outcomes are related in an interesting way, which we describe in this paper.

Collision problems such as these can be solved completely by applying the laws of conservation of energy and linear momentum (1-D or 2-D). During the time of impact, each ball exerts a normal (radial) force on the other.[1] Through the action of these forces, momentum and kinetic energy are exchanged between the two balls. The normal force on m_1 (F_1) and that on m_2 (F_2) must be equal and opposite because they belong to the same action-reaction pair.[2] In the head-on case, the normal forces must lie on the line connecting the centers of the balls (Fig. 3).

The oblique case is different—but not so much, actually. Resolve the incident velocity **v** in this case into two vector components (see Fig. 4): one in the direction of F_2 (the // direction) and the other perpendicular to the normal forces (the $\perp$ direction).

We label the former component as $\mathbf{v}_{//}$ and the later as $\mathbf{v}_{\perp}$. Note that the two normal forces and the velocity $\mathbf{v}_{//}$ appear exactly as they would in a head-on collision.[3] This means that after the collision, m_2 moves off with velocity $\mathbf{v}_{//}$ while the // component of m_1's velocity is reduced to zero. On the other hand, the velocity component $\mathbf{v}_{\perp}$ of m_1 remains unaltered during the collision (there is no force component in the $\perp$



Fig, 1. A head-on elastic collision with $m_1 = m_2$.

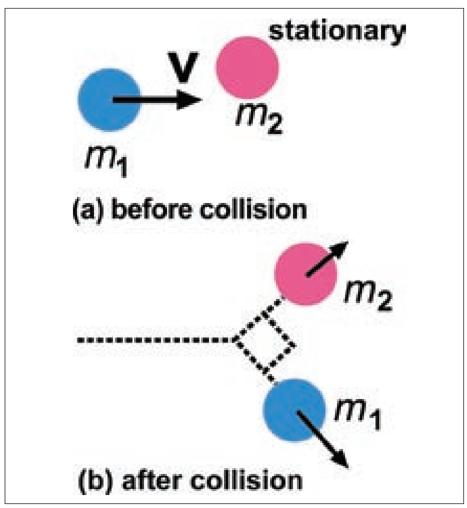


Fig. 2. An oblique elastic collision with $m_1 = m_2$.

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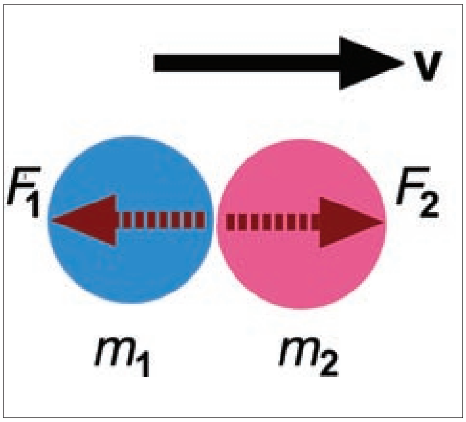

Fig. 3. In a head-on collision, the normal forces and the incident velocity v are co-linear.

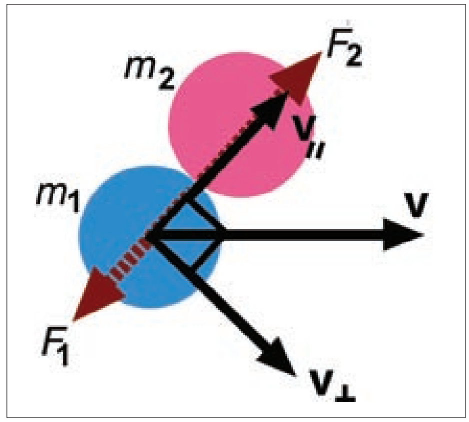

Fig. 4. Resolution of the incident velocity v into two perpendicular components.

direction). Since m_2 moves off in the // direction and m_1 in the $\perp$ direction, the paths of the two balls are perpendicular to each other. Notice that $v^2 = v_{//}^2 + v_\perp^2$ so $\frac{1}{2}m_1v^2 = \frac{1}{2}m_2v_{//}^2 + \frac{1}{2}m_1v_\perp^2$, ensuring that kinetic energy is always conserved during the collision.

The conceptual argument presented here is short and simple. It is helpful to students who have learned about head-on collisions and are beginning to study oblique ones.

References

1. We neglect all nonconservative forces, hence the total kinetic energy is conserved.
2. In reality, the objects are not perfectly rigid, so the "point of contact" between them may actually be an area. The "normal forces" in our discussion are spatial averages.
3. If we view the collision from a reference frame that is moving with constant velocity $\mathbf{v}_\perp$, what we see is a head-on collision in the // direction between m_1 (which has initial velocity $\mathbf{v}_{//}$) and m_2, which is initially at rest in that direction.

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In Principle, the Problems Are All Solved

"In 1926, Erwin Schrödinger first derived the analytical solution for the electronic states of the hydrogen atom.[1] Not long after this, Paul Dirac said: 'The underlying physical laws necessary for the mathematical treatment of a large part of physics and the whole of chemistry are thus completely known, and the difficulty is only that the exact application of these laws leads to equations much too complicated to be soluble. It therefore becomes desirable that approximate practical methods of applying quantum mechanics should be developed which can lead to an explanation of the main features of complex atomic systems without too much computation.'[2] "[3]

1. E. Schröedinger, *Ann. Phys.* **79**, 361 (1926).
2. P.A.M. Dirac, *Proc. Roy. Soc. London,* Ser. A **123**, 714 (1929).
3. David C. Clary, *Sci.* **314**, 265-266 (Oct. 13, 2006).