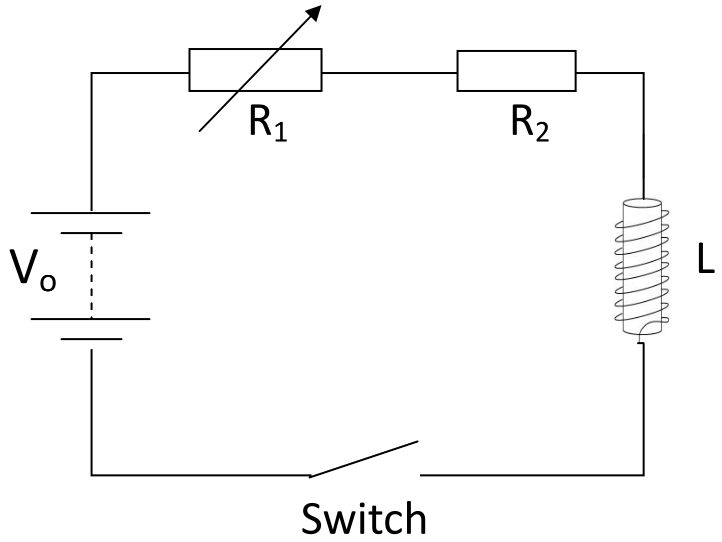
Growth of current in an RL d.c. circuit



In the above circuit, the two resistors obey Ohm's law, i.e.

V = IR.

The inductor L is assumed to have no resistance, it will generate a back emf whenever there is a change in the current passing through it, $\epsilon = -L\frac{dI}{dt}$

Around the circuit, total emf = total p.d. Hence,

$V_o + \epsilon = I(R_1 + R_2)$ or

$$V_o - L\frac{dI}{dt} = I(R_1 + R_2) \quad \text{..............(1)}$$

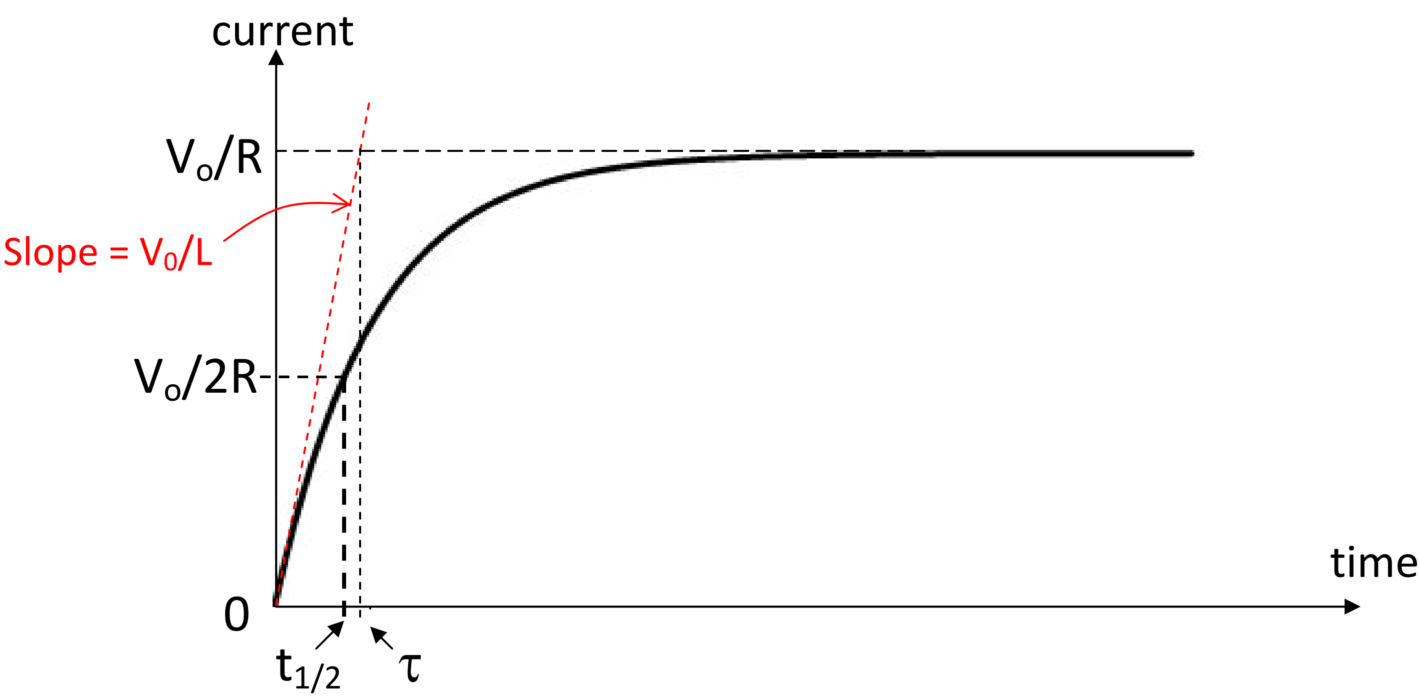
Eq (1) leads us to the solution of the current as a function of time t.

With the initial condition I = 0 at t = 0, Eq (1) is solved

$$I = \frac{V_o}{R}(1 - e^{-\frac{t}{\tau}}) \quad \text{.....................(2)}$$

Where R = R_1+ R_2 is the total resistance and $\tau = \frac{L}{R}$ is called the time constant of the LR circuit.

According to Eq (2), we get the graph of I-t.



- The current **rises exponentially** and eventually up to the steady value V_0/R. Inductance L can be regarded as a kind of inertia, it hinders any changes of current in the circuit.

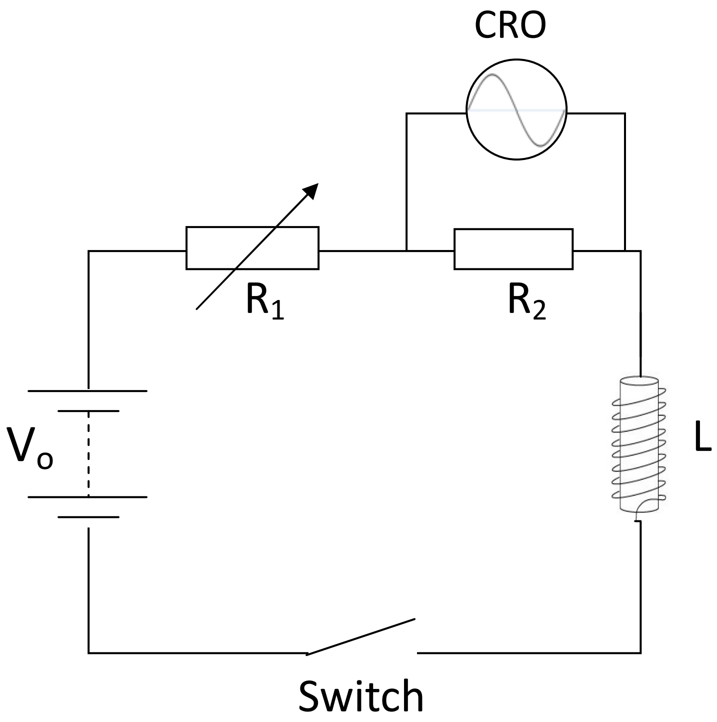
- Putting I =0 (the current at t = 0) in Eq (1), we obtain the initial rate of rise of current, dI/dt = V_0/L . Thus, the red dotted line shown in the above graph has the slope V_0/L.

- The time $t_{1/2}$ is the half-life, it is the time for the current rising to half of its steady value V_o/R. Mathematically,

$$t_{1/2} = \tau \ln 2 = \frac{L}{R}\ln 2$$

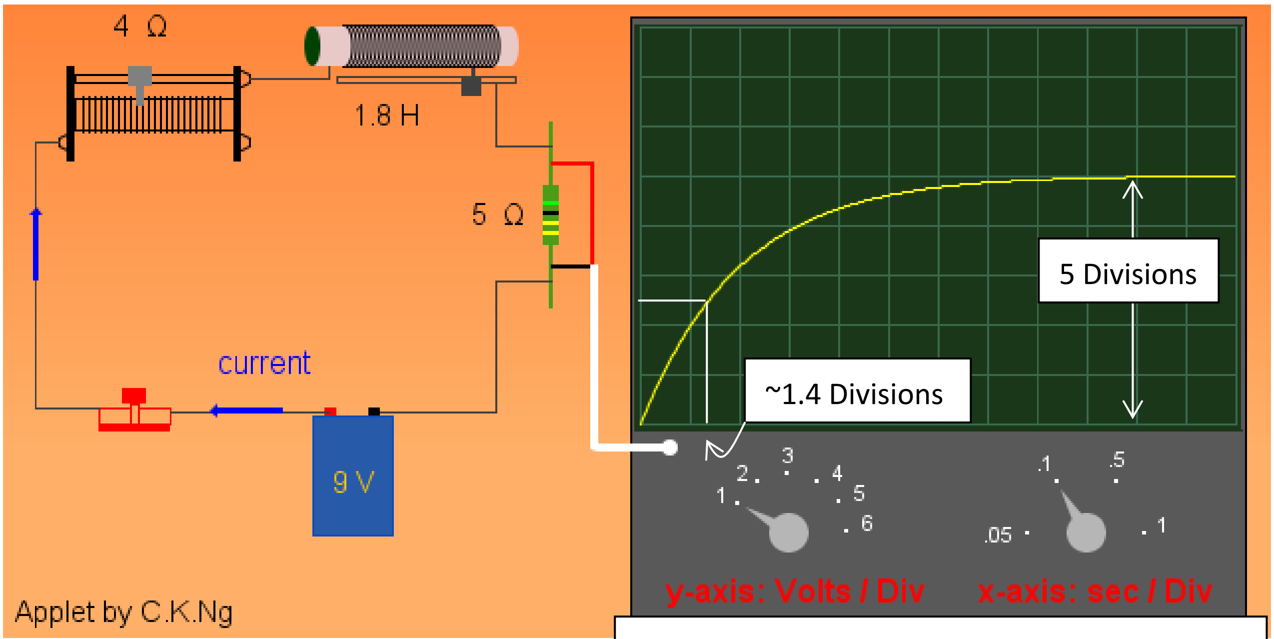
- The time constant τ (or half life $t_{1/2}$) gives us a sense on how long the transient stage is about to last. If L = 0, the current will build up to the steady value immediately after the switch is closed.

- As a CRO is used to monitor the time-varying current, we can connect it in this way:



The trace displayed on the CRO is therefore the voltage across R_2. Nevertheless, the trace is absolutely identical to the current-time graph, except that the final steady value is now replaced by $V_o(\frac{R_2}{R_1+R_2})$. The exponential curve, the time constant and the half-life are all the same.

Example



http://ngsir.netfirms.com/englishhtm/RL.htm

Calculations:

1. On the CRO, Steady value = 5 Divisions, which corresponds to 5 Div X 1 V/Div = 5V.

   Theoretically, the voltage of battery 9V is finally divided between the 4Ω and the 5Ω resistors. The part shared by the 5Ω resistor is 9X5/(5+4) = 5V.

2. On the CRO, the half-life is observed to be ~1.4 Div. It corresponds to 1.4 Div X 0.1sec/Div = 0.14s.

   Theoretically, $t_{1/2} = \tau \ln 2 = \frac{L}{R}\ln 2 = \frac{1.8}{4+5}\ln 2 = 0.138$s.